GNU Octave Interval Package

First Anniversary
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Outline

- Why am I doing this?
- Interval arithmetic
- New features from IEEE Std 1788-2015, IEEE standard for interval arithmetic
- Package development @ Octave Forge
Why am I doing this?

- Fun
- Freedom
- Learn
- Create & Share
What is interval arithmetic?

- No approximate results, but boundaries
- Guaranteed enclosures of the exact result
- Function evaluation on a range of possible values

```matlab
>> sin(pi)
an = 1.2246e-16
>> sin(infsup("pi"))
an = [-3.2163e-16, +1.2247e-16]
>> cos(infsup([-2, +1]))
an = [-0.41615, +1]
```
Interval arithmetic is always correct

.. and optionally accurate

\[ x \in X \Rightarrow f(x) \in f(X) \]

>> x = rand (100, 1);
>> X = infsup (x);
>> sum (ismember (... csc (x), csc (X)))
ans = 98
**Brouwer fixed-point theorem**

\[ f(X) \subseteq X \Rightarrow f(x_0) = x_0 \text{ for at least one } x_0 \in X \]

- **Continuous interval function**
- **Bounded interval**

```plaintext
>> X = infsupdec("[-1, +1]";
>> cos(X)
ans = [0.5403, 1] com
>> subset(ans, X)
ans = 1
```

Interval “decoration” proves that the function is continuous.
Interval methods are superior

- For example: Bisection

- Find enclosures of all roots within a given interval

- Use interval Newton method if derivative is known

```plaintext
>> fzero (@cos, 0)
ans = 1.5708
>> fzero (@cos, ...
    infsup (["[-2, +5]"])
ans = 3x1 interval vector
[-1.5708, -1.5707]
[1.5707, 1.5708]
[4.7123, 4.7124]
```
Brand-new IEEE Std 1788-2015

- Interchange formats and constructors
- Semantics for evaluation
  - Empty sets
  - Limit values
  - Exceptions
- List of required operations

Initiating the Project
- Mobilizing the Working Group
- Drafting the Standard
- Balloting the Standard
- Gaining Final Approval
- Maintaining the Standard

June 11

Octave has the first implementation!

IEEE standard life-cycle
The standard demands reverse mode operations

\[ f(x) = y \]
\[ x \in X_0, \ y \in Y \]

\[ X_1 = f\text{Rev}(Y, X_0) \]

interval constraints

\[
\gg \text{tanrev} (\"[.25, .75]\", \ldots \\
2 * \text{infsup} (\"[0, pi]\"))
\]
\[
\text{ans} = [3.3865, 3.7851]
\]

improved solution
Intervals: the universal tool—not

- No free lunch: For exact math use symbol calculations
- Global error intervals might become meaningless: For large value ranges consider statistic methods
- Major obstacle: the “dependency problem” Can all possible values be taken independently?

\[ X^2 \subseteq X \cdot X \]
Unit tests are mandatory for verified libraries

- Traditional Octave `%! tests`
- Doctest package
- Collection of test cases from several free interval libraries
  - portable
  - domain-specific language
  - compiled into Octave test code and included in release

  ca. 700 test cases
  ca. 340 test cases (+60 in manual)
  ca. 9100 test cases from ... libieeep1788, MPFI, FILIB, C-XSC, and self-made

For comparison ...
  in all other packages: ca. 5700
  in core: ca. 13000
User feedback would be better than unit tests

- I did receive almost no user feedback whatsoever—are there users actually?
- Communication with distributors is difficult
  - They fix technical issues instead of reporting upstream
- Yesterday during code sprint
  - Useful feedback from user code
- Attempts to advertise
  - Octave mailing lists (little feedback)
  - Experts mailing lists (mentioned in conference)
  - Free software dictionary
  - Wikipedia: Interval arithmetic
  - Popular link collection
  - Links on related project websites
  - Other developers in the field (many abandoned projects by scientists)
There is amazing reach with Octave Forge

- Fresh Ports (FreeBSD)
- MXE-Octave (Windows)
- MacPorts (OS X)
- openSUSE
- (possibly one day) Debian
- Cygwin

and interested users on the mailing lists who use pkg install -forge...
Best practices from the interval package

- Texinfo is great
  - easy to learn (LaTeX dialect)
  - good looking documentation
  - easy to maintain
  - (PDF not so great)

- Makefile for developers
  - simplifies package release
  - simplifies build automation
    (Jenkins etc.)
  - essential for complex builds
    (code and doc generation)

- Package manual supplementary to function reference
  - better than federation of the two as used by Octave core
Current work queue—outlook is golden

- Multivariate interval Newton method
- Polynomial root finding, Eigenvalues
  - problem: complex values
- Utility functions and matrix functions
- Different plotting functions, colormaps, plotting of unbound intervals

- Basics are done
- Release cycles getting larger
  - some rest for Carne
Images by

Ruth García (Guy riding gnu)
http://www.gnu.org/graphics/umsa/
umsa

Frits Ahlefeldt-Laurvig (Newton)
https://www.flickr.com/photos/hikingartist/6217869031

Drake Emko & Jen Brodzik
(Hackless comic)
http://hackles.org/cgi-bin/archives.pl?request=334

Several others
Wikipedia and own work